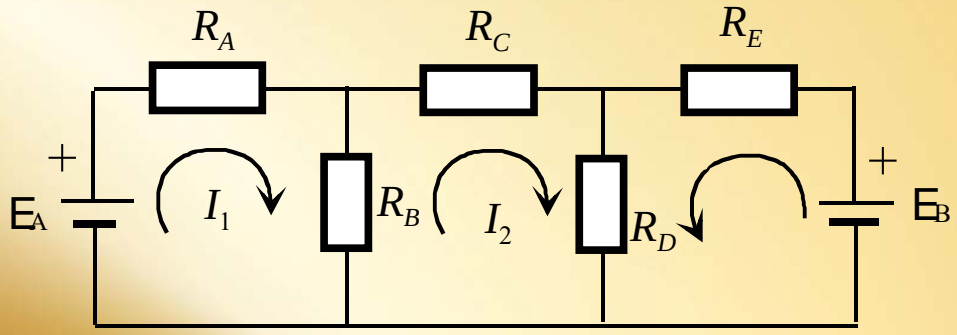
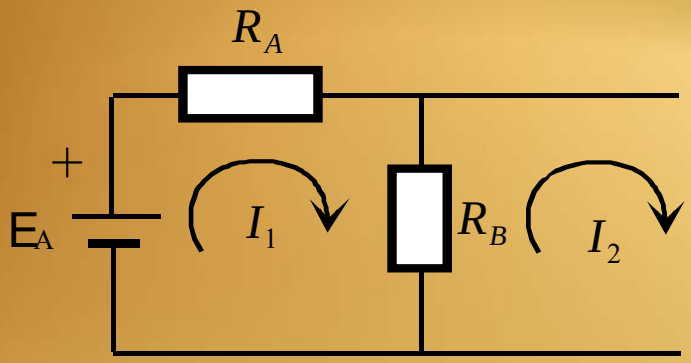
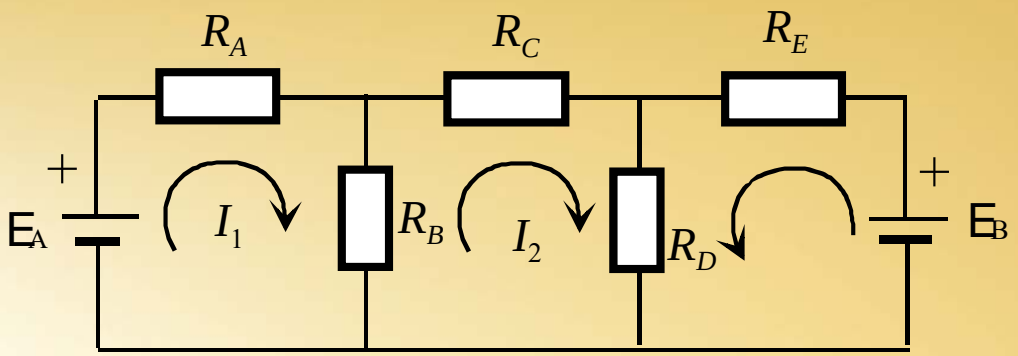
3.18 Rešavanje složenih električnih kola metodom konturnih struja





$$I_1 R_A + (I_1 - I_2) R_B = E_A$$



$$(I_2 - I_1)R_B + I_2 R_C + (I_2 + I_3)R_D = 0$$

$$(I_3 + I_2)R_D + I_3 R_E = \mathcal{E}_B$$

$$I_1 R_A + (I_1 - I_2) R_B = \mathsf{E}_A$$

$$(I_2 - I_1) R_B + I_2 R_C + (I_2 + I_3) R_D = 0$$

$$(I_3 + I_2) R_D + I_3 R_E = \mathsf{E}_B$$

$$\begin{aligned}
(R_A + R_B) I_1 \quad & - R_B I_2 \quad & + 0 I_3 \quad & = \mathsf{E}_A \\
- R_B I_1 \quad & + (R_B + R_C + R_D) I_2 \quad & + R_D I_3 \quad & = 0 \\
0 I_1 \quad & + R_D I_2 \quad & + (R_D + R_E) I_3 \quad & = \mathsf{E}_B
\end{aligned}$$

$$R_{11}I_1 + R_{12}I_2 + R_{13}I_3 = \mathsf{E}_1$$

$$R_{21}I_1 + R_{22}I_2 + R_{23}I_3 = \mathsf{E}_2$$

$$R_{31}I_1 + R_{32}I_2 + R_{33}I_3 = \mathsf{E}_3$$

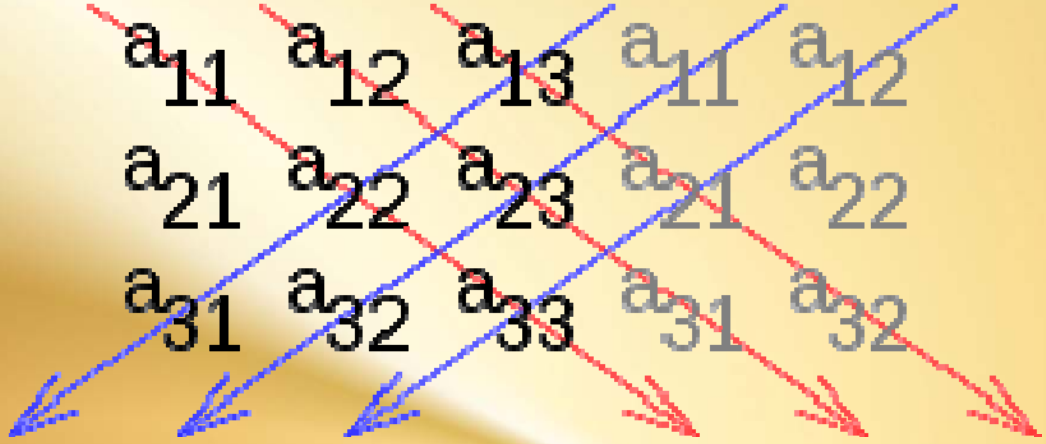
- Sopstvena otpornost prve konture $R_{11} = R_A + R_B$
- Sopstvena otpornost druge konture $R_{22} = R_B + R_C + R_D$
- Sopstvena otpornost treće konture $R_{33} = R_D + R_E$

$$R_{12} = R_{21} \qquad R_{13} = R_{31} = 0$$

$$R_A = 2\Omega \quad R_B = 3\Omega \quad R_C = 4\Omega \quad R_D = 5\Omega \quad R_E = 6\Omega$$

$$E_1 = 4{,}5\text{V} \qquad E_2 = 9\text{V}$$

- Sarusovo pravilo



- -

+

$$\Delta = \begin{vmatrix} 5 & -3 & 0 \\ -3 & 9 & 5 \\ 0 & 5 & 11 \end{vmatrix} \begin{matrix} 5 & -3 \\ -3 & 9 \\ 0 & 5 \end{matrix} = 224$$

$$\Delta_1 = \begin{vmatrix} 4{,}5 & -3 & 0 \\ 0 & 9 & 5 \\ 9 & 5 & 11 \end{vmatrix} \begin{matrix} 4{,}5 & -3 \\ 0 & 9 \\ 9 & 5 \end{matrix} = 198$$

$$I_1 = \frac{\Delta_1}{\Delta} = 0{,}88\text{A}$$

$$\Delta_2 = \begin{vmatrix} 5 & 4{,}5 & 0 \\ -3 & 0 & 5 \\ 0 & 9 & 11 \end{vmatrix} \begin{matrix} 5 & 4{,}5 \\ -3 & 0 \\ 0 & 9 \end{matrix} = -0{,}34\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = -0{,}34\text{A}$$

$$\Delta_3 = \begin{vmatrix} 5 & -3 & 4{,}5 \\ -3 & 9 & 0 \\ 0 & 5 & 9 \end{vmatrix} \begin{matrix} 5 & -3 \\ -3 & 9 \\ 0 & 5 \end{matrix} = 256{,}5$$

$$I_3 = \frac{\Delta_3}{\Delta} = 1{,}14\text{A}$$

$$I_A = I_1 = 0{,}88\text{A}$$

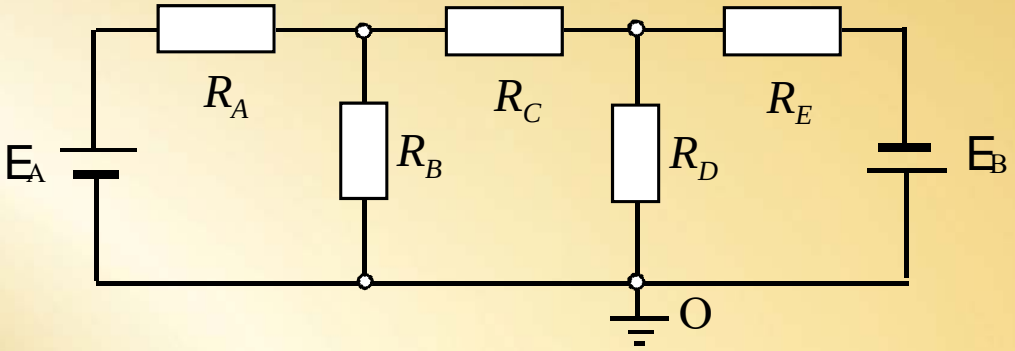
$$I_B = I_1 - I_2 = 1{,}22\text{A}$$

$$I_C = I_2 = -0{,}34\text{A}$$

$$I_D = I_2 + I_3 = 0{,}8\text{A}$$

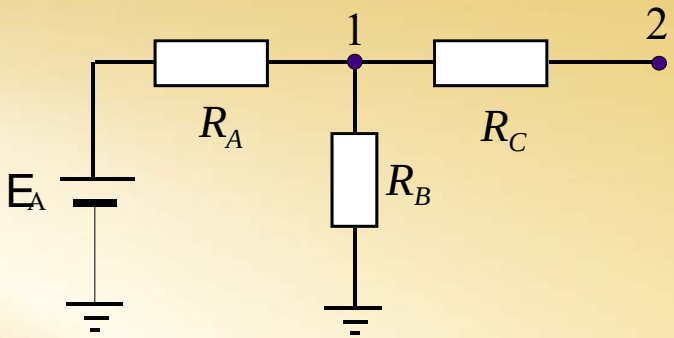
$$I_E = I_3 = 1{,}14\text{A}$$

3.19 Rešavanje složenih električnih kola metodom potencijala čvorova

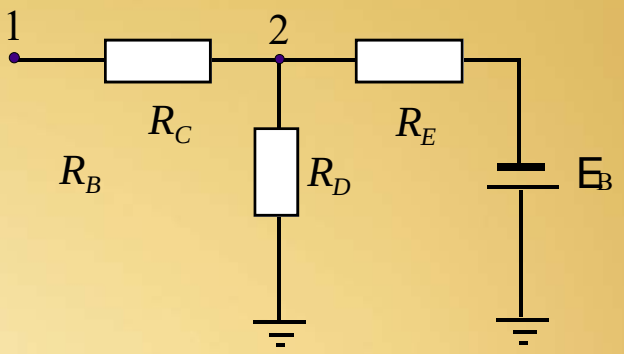


$$\frac{U_{10}-V_A}{R_A}+\frac{U_{10}}{R_B}+\frac{U_{10}-U_{20}}{R_C}=0$$

$$\frac{U_{20}-U_{10}}{R_C}+\frac{U_{20}}{R_D}+\frac{U_{20}-V_E}{R_E}=0$$



a



b

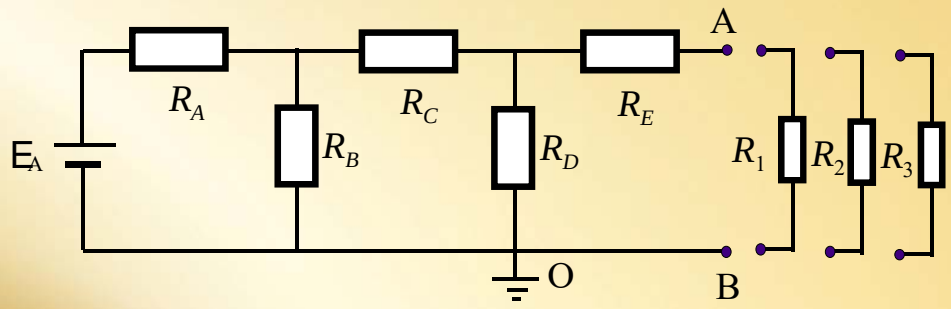
$$\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)U_{10}-\frac{1}{R_C}U_{20}=\frac{1}{R_A}V_A$$

$$-\frac{1}{R_C}U_{10}+\left(\frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E}\right)U_{20}=-\frac{1}{R_E}V_E$$

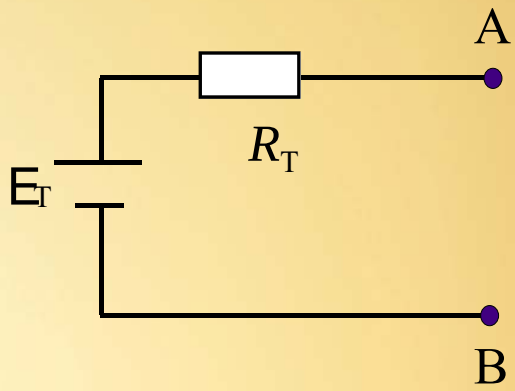
$$\left(G_A + G_B + G_C\right)U_{10} - G_C U_{20} = G_A V_A$$

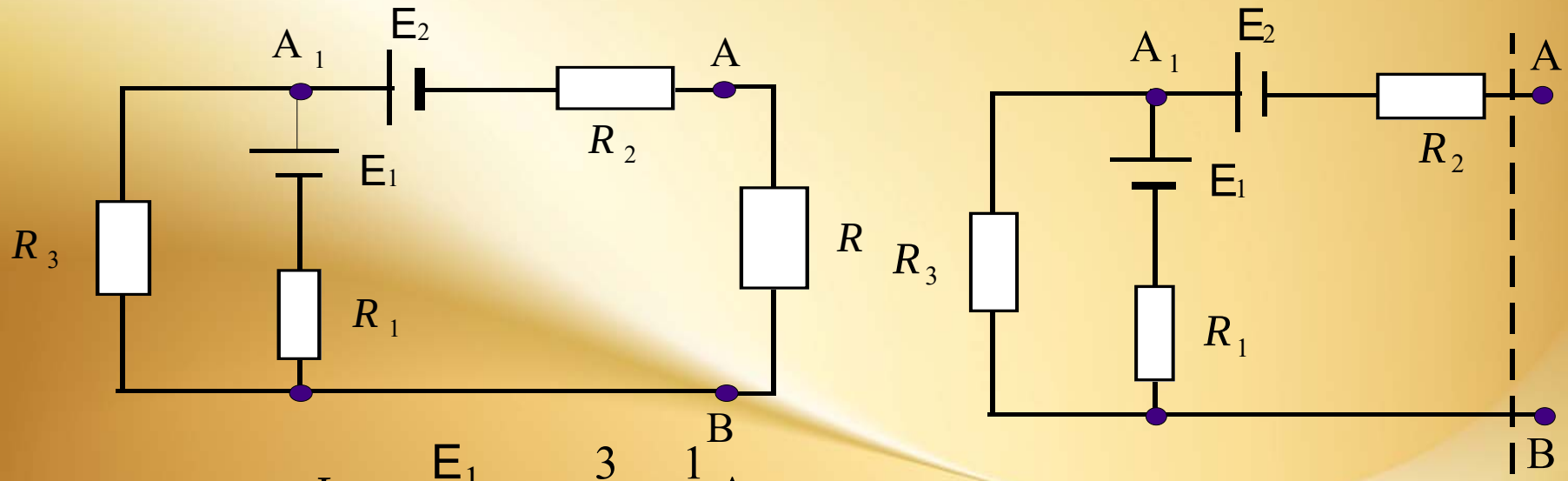
$$-G_C U_{10} + \left(G_C + G_D + G_E\right)U_{20} = -G_E V_E$$

3.20 Ekvivalentna električna kola



3.21 Tevenenova teorema

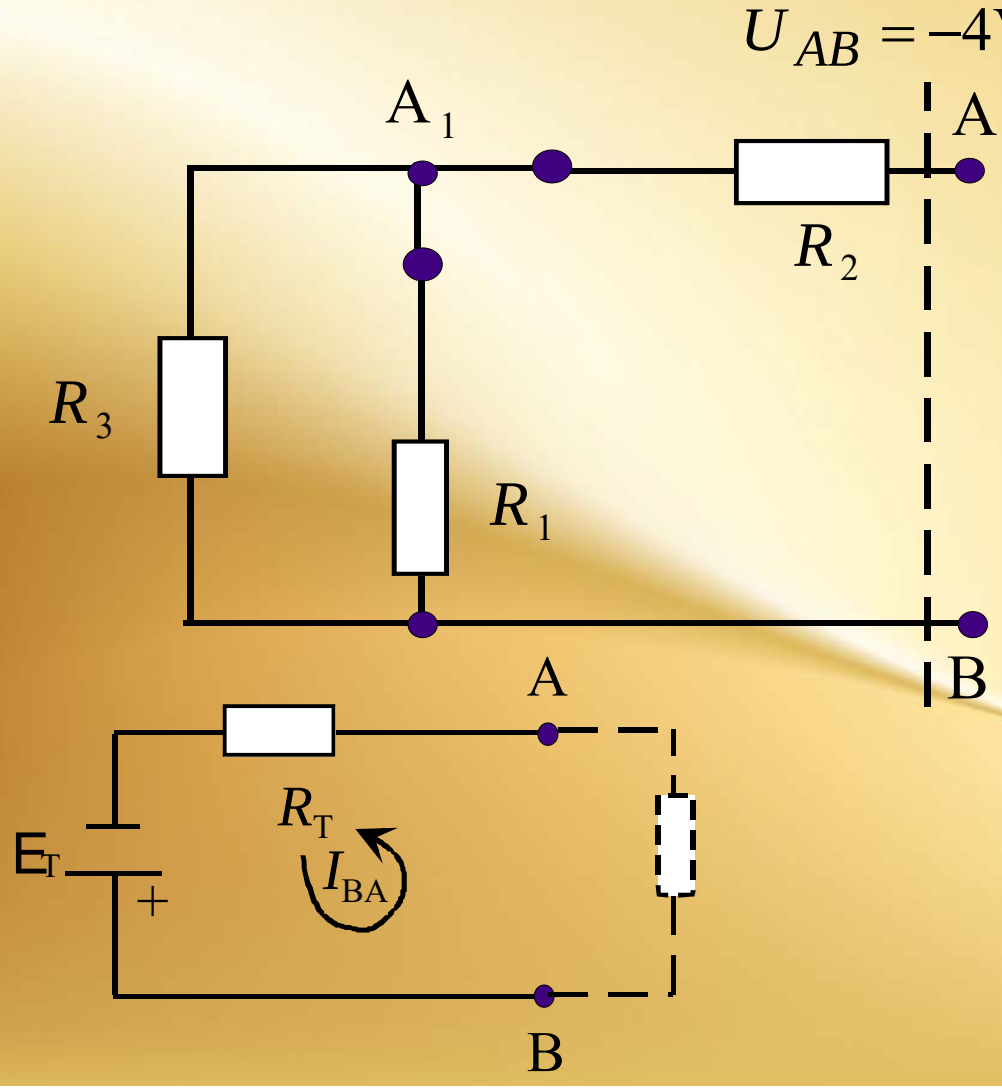




$$I = \frac{\mathsf{E}_1}{R_1 + R_3} = \frac{3}{15} = \frac{1}{5}\,\mathrm{A}$$

$$\mathsf{E}_1 = U_{A_1B} + R_1 I \Rightarrow U_{A_1B} = \mathsf{E}_1 - I \cdot R_1 = 2\,\mathrm{V}$$

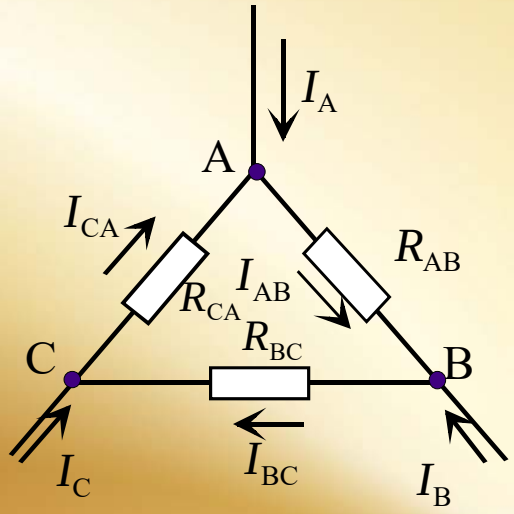
$$\mathsf{E}_2 = U_{A_1B} + U_{BA} \Rightarrow U_{BA} = \mathsf{E}_2 - U_{A_1B} = 4\text{V}$$



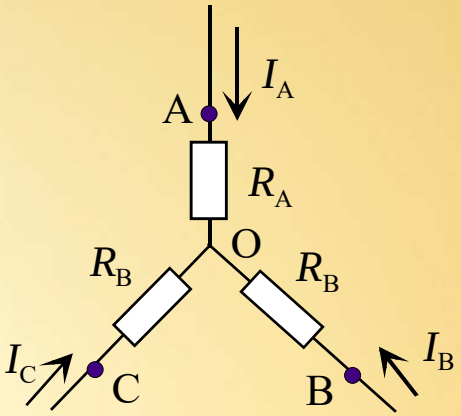
$$R_T = \frac{R_1 \cdot R_3}{R_1 + R_3} + R_2 = \frac{22}{3}\Omega$$

$$I_{BA} = \frac{\mathsf{E}_T}{R_T + R} = \frac{6}{41}\text{A}$$

3.22 Transformacije u električnim kolima



a



b

$$R_{AB}I_{AB} + R_{BC}I_{BC} + R_{CA}I_{CA} = 0$$

$$-I_A + I_{AB} - I_{CA} = 0$$

$$-I_B + I_{BC} - I_{AB} = 0$$

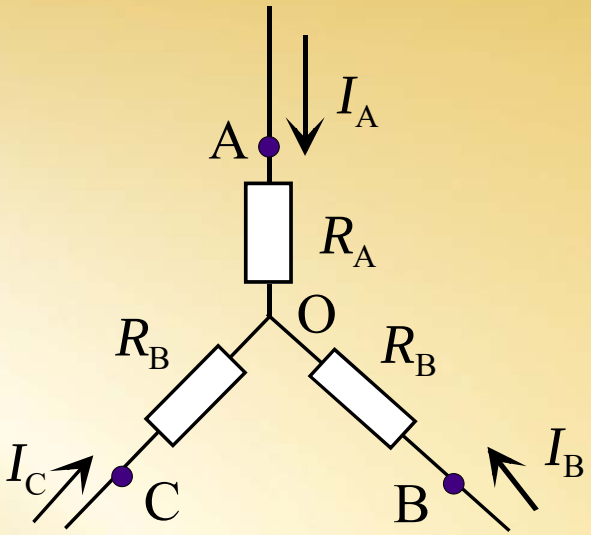
$$I_{CA} = -I_A + I_{AB}$$

$$I_{BC} = I_B + I_{AB}$$

$$R_{AB} I_{AB} + R_{BC} I_{BC} + R_{CA} I_{CA} = 0$$

$$I_{AB} = \frac{R_{CA} I_A - R_{BC} I_B}{R_{AB} + R_{BC} + R_{CA}}$$

$$U_{AB} = R_{AB} I_{AB} = \frac{R_{AB} R_{CA} I_A - R_{AB} R_{BC} I_B}{R_{AB} + R_{BC} + R_{CA}}$$



b

$$U_{AB} = R_A I_A - R_B I_B$$

$$U_{AB} = R_{AB} I_{AB} = \frac{R_{AB} R_{CA} I_A - R_{AB} R_{BC} I_B}{R_{AB} + R_{BC} + R_{CA}}$$

$$\frac{R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}}I_A-\frac{R_{AB}R_{BC}}{R_{AB}+R_{BC}+R_{CA}}I_B=R_AI_A-R_BI_B$$

$$R_A=\frac{R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$R_B=\frac{R_{AB}R_{BC}}{R_{AB}+R_{BC}+R_{CA}}$$

$$R_C=\frac{R_{BC}R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_{BC} = R_{AB} \frac{R_C}{R_A}$$

$$R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_{CA} = R_{AB} \frac{R_C}{R_B}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A \left(R_{AB} + R_{BC} + R_{CA} \right) = R_{AB} R_{CA}$$

$$R_{BC} = R_{AB} \frac{R_C}{R_A}$$

$$R_{CA} = R_{AB} \frac{R_C}{R_B}$$

$$R_A \left[R_{AB} + R_{AB} \frac{R_C}{R_A} + R_{AB} \frac{R_C}{R_B} \right] = R_{AB}^2 \frac{R_C}{R_B}$$

$$R_A\left[1+\frac{R_C}{R_A}+\frac{R_C}{R_B}\right]=R_{AB}\frac{R_C}{R_B}$$

$$R_{AB}=R_A+R_B+\frac{R_AR_B}{R_C}$$

$$R_{BC}=R_B+R_C+\frac{R_BR_C}{R_A}$$

$$R_{CA}=R_C+R_A+\frac{R_CR_A}{R_B}$$

Primer Ako su poznate otpornosti otpornika vezanih u trougao: $R_{AB} = 100\Omega$, $R_{BC} = 200\Omega$ i $R_{CA} = 300\Omega$ koliko iznose ekvivalentne otpornosti za vezanih u zvezdu?

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{100\Omega 300\Omega}{100\Omega + 200\Omega + 300\Omega} = \frac{300}{6} = 50\Omega$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \frac{100\Omega 200\Omega}{100\Omega + 200\Omega + 300\Omega} = \frac{200}{6} = \frac{100}{3} = 33{,}33\Omega$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{200\Omega 300\Omega}{100\Omega + 200\Omega + 300\Omega} = \frac{600}{6} = 100\Omega$$

Ako su poznate otpornosti otpornika vezanih u zvezdu:

$R_A = 50\Omega$, $R_B = \frac{100}{3}\Omega$ i $R_C = 100\Omega$,

koliko iznose ekvivalentne otpornosti za vezu u trougao?

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} = 50\Omega + \frac{100}{3}\Omega + \frac{50\Omega\frac{100}{3}\Omega}{100\Omega} = \frac{15000\Omega + 10000\Omega + 5000\Omega}{300\Omega}$$

$$R_{AB} = \frac{30000}{300} = 100\Omega$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} = \frac{100}{3}\Omega + 100\Omega + \frac{\frac{100}{3}\Omega 100\Omega}{50\Omega} = 200\Omega$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B} = 100\Omega + 50\Omega + \frac{100\Omega 50\Omega}{\frac{100}{3}\Omega} = 300\Omega$$

Priprema za I kolokvijum

- 3. Koliko je rastojanje na kome treba da se nalaze dva naelektrisana tela u vodi relativne dielektrične propustljivosti 81 da bi sila koja između njih deluje bila ista kao u vakuumu na rastojanju 18 cm?

$$F_1 = \frac{k_0}{\varepsilon_r}\frac{q_1 q_2}{x^2}$$

$$F_1 = \frac{k_0}{1}\frac{q_1 q_2}{r^2}$$

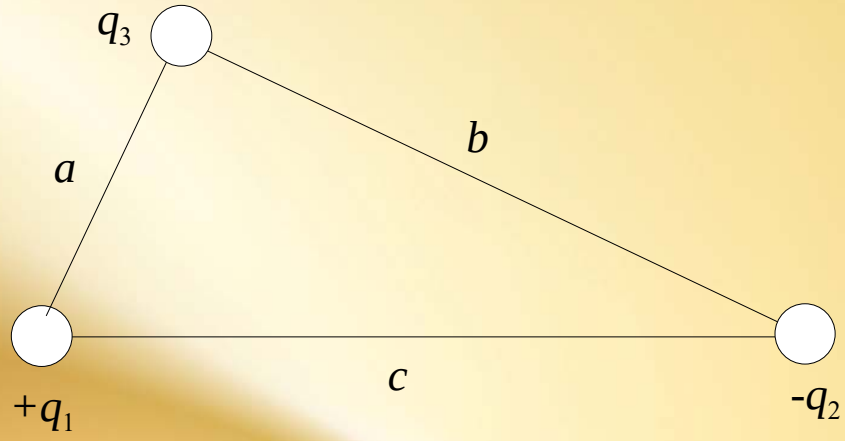
$$F_1 = F_2$$

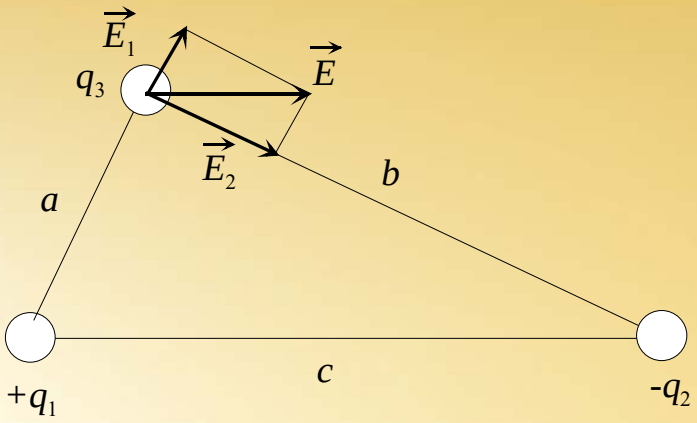
$$\frac{k_0}{\varepsilon_r}\frac{q_1 q_2}{x^2} = \frac{k_0}{1}\frac{q_1 q_2}{r^2}$$

$$\varepsilon_r x^2 = r^2$$

$$x = 2\text{cm}$$

2. Dva tačkasta naeektrisanja 6,7nC i -13,2nC nalaze se u vazduhu na međusobnom rastojanju 5cm. Odrediti jačinu električnog polja u tački koja je udaljena 3cm od pozitivnog i 4cm od negativnog naelektrisanja.



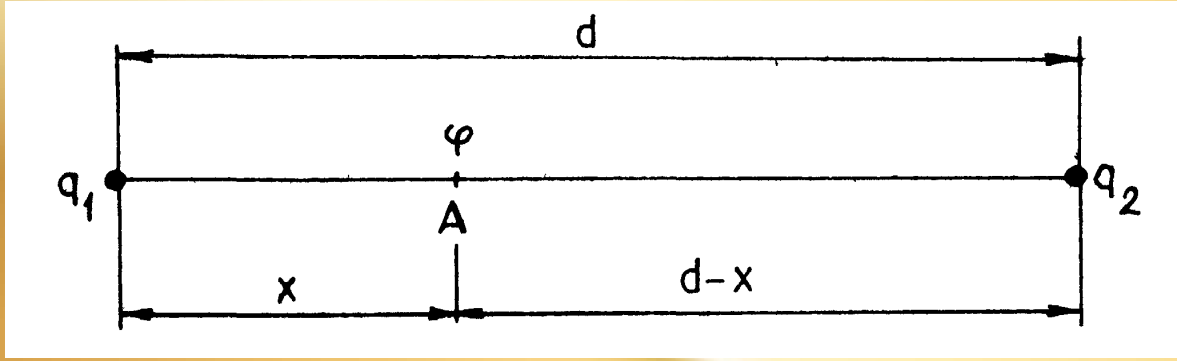


$$E_1 = \frac{k_0}{\varepsilon_r}\frac{q_1}{a^2}$$

$$E_1 = \frac{k_0}{\varepsilon_r}\frac{q_2}{b^2}$$

$$E = \sqrt{E_1^2 + E_2^2} = 101\frac{\mathrm{kN}}{\mathrm{C}}$$

2. Negativno naelektrisano telo ima, po apsolutnoj vrednosti, pet puta veću količinu naelektrisanja od drugog pozitivno naelektrisanog tela. Tela se nalaze na rastojanju 72 cm. Na duži koja ih spaja, odrediti tačku u kojoj je potencijal nula.



$$\varphi_1 = \frac{k_0}{\varepsilon_r}\frac{q_1}{x}$$

$$\varphi_2 = -\frac{k_0}{\varepsilon_r}\frac{5q_1}{d-x}$$

$$\varphi = 0$$

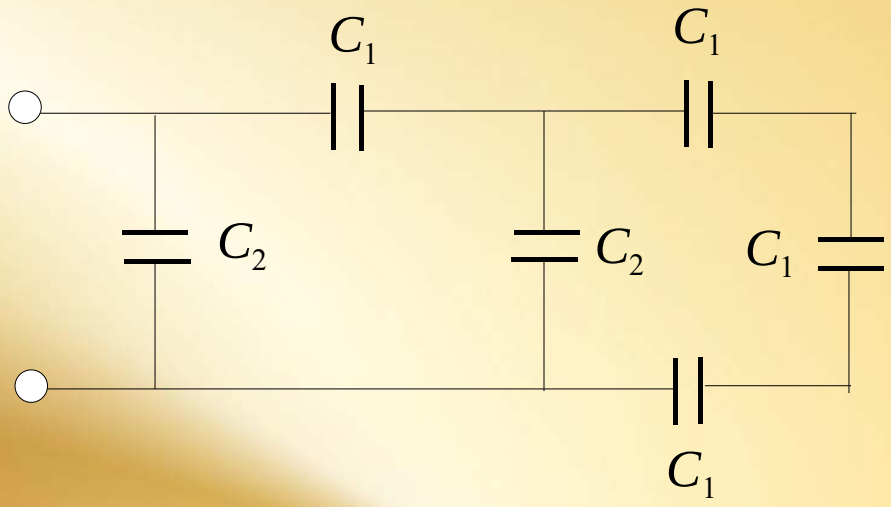
$$0 = \frac{k_0}{\varepsilon_r}\frac{q_1}{x} - \frac{k_0}{\varepsilon_r}\frac{5q_1}{d-x}$$

$$\frac{1}{x} = \frac{5}{d-x}$$

$$x = \frac{1}{6}d = 12\text{cm}$$

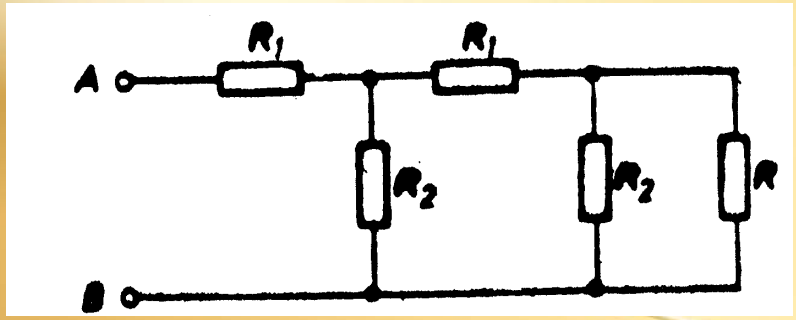
Odrediti ekvivalentni kapacitet baterije kondenzatora prikazane na slici ako je

$C1=2\mu F$ i $C2=1\mu F$.



$$C_e = C_2 + \frac{C_1(3C_2 + C_1)}{3C_2 + 4C_1} = 1{,}9\mu\text{F}$$

- Ekvivalentna otpornost između tačaka A i B na slici treba da bude jednaka otpornosti $2R_1$. Ako je
- $R_1 = 100\Omega$, $R_2 = 200\Omega$, naći otpornost *R*.



$$R_e = \frac{R_2\left(R_1 + \dfrac{R_2 R}{R_2 + R}\right)}{R_2 + R_1 + \dfrac{R_2 R}{R_2 + R}} = 2R_1$$

$$R = \frac{R_1^2 R_2}{R_2^2 - R_1^2 - R_1 R_2} = 200\Omega$$

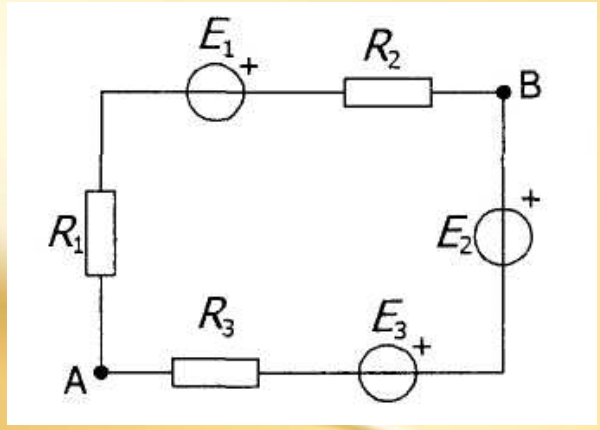
Elektromotorna sila izvora struje je 6V, unutrašnji otpor izvora je 0,5Ω, a otpor spoljašnjeg dela kola je 11,5Ω. Kolika je jačina struje u kolu, a koliki napon na polovima izvora?

$$I = \frac{\mathrm{E}}{r + R} = 0{,}5\mathrm{A}$$

$$U = RI = 5{,}75\mathrm{V}$$

Generatori $\mathcal{E}_1$ = 10 V, $\mathcal{E}_2$ = 20 V i $\mathcal{E}_3$ = 30 V, zanemarljivih unutrašnjih otpornosti, i otpornici R_1 = 15 Ω, R_2 = 5 Ω i R_3 = 20 Ω povezani su kao na slici. Odrediti:

1. intenzitet struje u kolu,
2. napon između tačaka A i B,

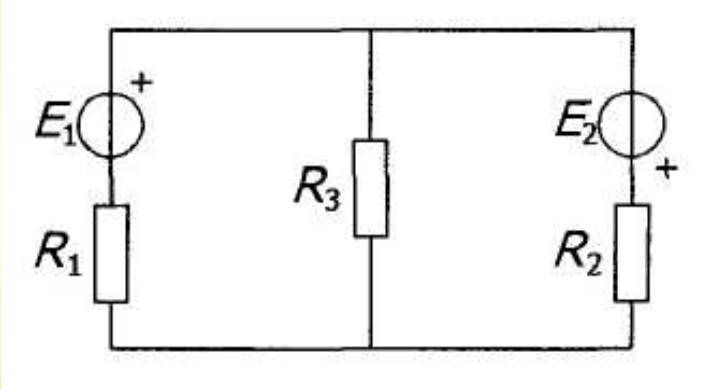


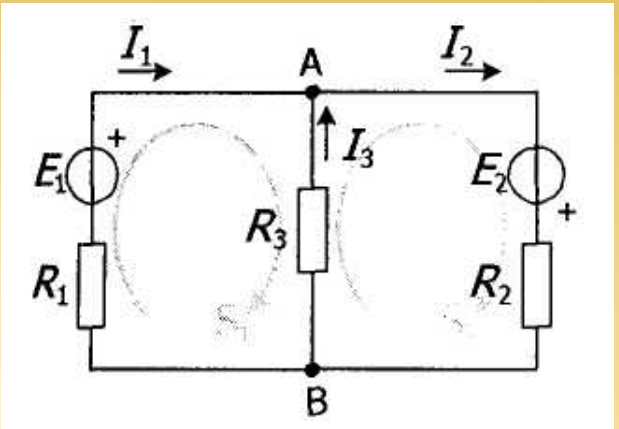
$$I = \frac{E_1 - E_2 - E_3}{R_1 + R_2 + R_3} = -1\text{A}$$

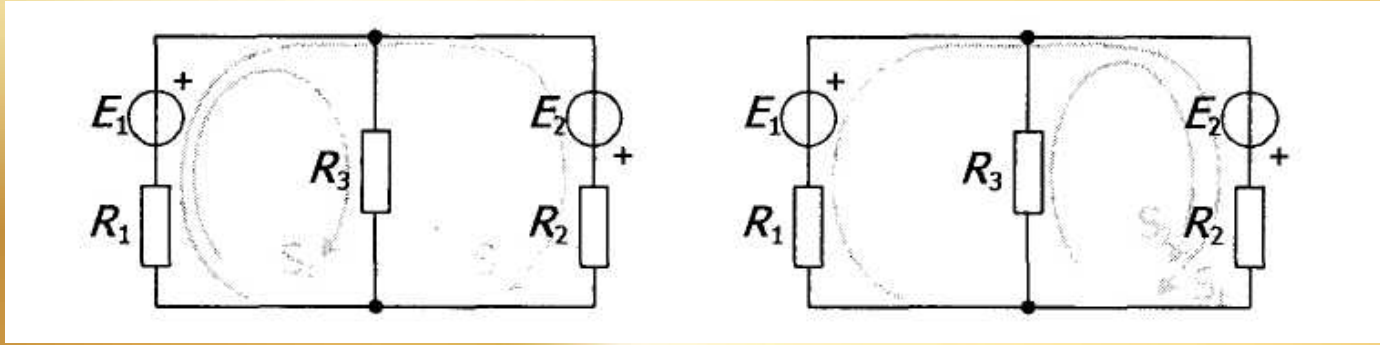
$$U_{AB} = R_2 I - E_1 + R_1 I = -30V$$

$$U_{AB} = -E_2 - E_3 - R_3 I = -30V$$

Za kolo prikazano na slici odrediti intenzitete struja u svim granama neposrednom primenom Kirhofovih zakona, ako je $E_1 = 6$ V, $E_2 = 20$ V i $R_1 = 700\ \Omega$, $R_2 = 300\ \Omega$, $R_3 = 400\Omega$.







$$I_1 = I_2 - I_3$$

$$E_1 = R_1 I_1 - R_3 I_3$$

$$E_2 = R_2 I_2 + R_3 I_3$$

$$-I_1 + I_2 - I_3 = 0$$

$$700I_1 - 400I_3 = 6$$

$$300I_2 + 400I_3 = 20$$

$$I_3 = -I_1 + I_2$$

$$1100I_1 - 400I_2 = 6$$

$$-400I_1 + 700I_2 = 20$$

$$\Delta = \begin{vmatrix} 1100 & -400 \\ -400 & +700 \end{vmatrix} = 770000 - 160000 = 610000$$

$$\Delta_1 = \begin{vmatrix} 6 & -400 \\ 20 & 700 \end{vmatrix} = 4200 + 8000 = 12200$$

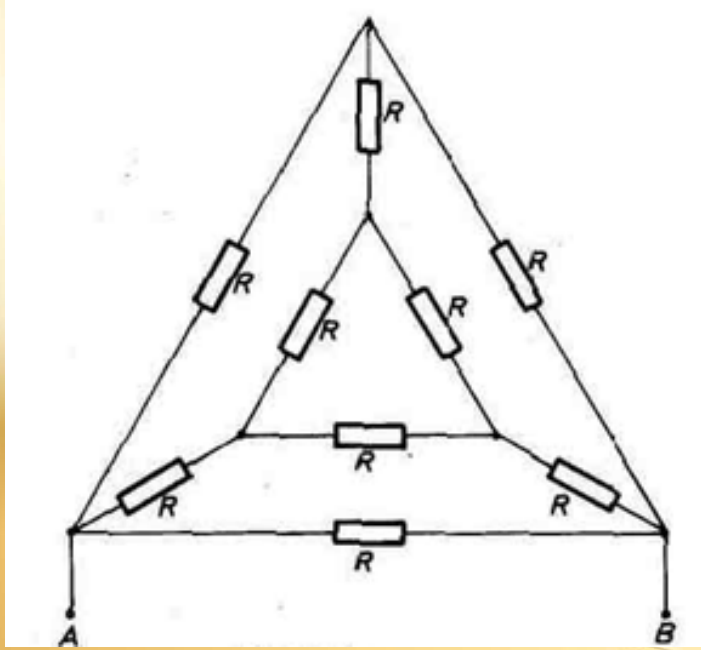
$$\Delta_2 = \begin{vmatrix} 1100 & 6 \\ -400 & 20 \end{vmatrix} = 22000 + 2400 = 24400$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{12200}{610000} = 0{,}02\text{A} = 20\text{mA}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{24400}{610000} = 0{,}04\text{A} = 40\text{mA}$$

$$I_3 = -I_1 + I_2 = 20\text{mA}$$

Odrediti ekvivalentnu otpornost između tačaka A i B ako je $R = 150\Omega$



$$R_{AB} = 80\Omega$$